RADIOSITY BASED MODEL FOR TERRAIN EFFECTS ON MULTI-ANGULAR VIEWS

Christoph C. Borel and Siegfried A.W. Gerstl

Los Alamos National Laboratory

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CONTENTS

Introduction

Problem:

In mountainous terrain, reflections from adjacent slopes can change the apparent reflectance of a pixel.
see e.g. Dozier and Frew (1989), Proy, Tanre and Deschamps (1989), Kawata et al. (1992)

Other Terrain Effects :

- effects of slope angles
- terrain height effects on the path radiance

Goals :

- get more insight into the effect of reflected light from adjacent terrain
- simple wedge model includes multiple reflections

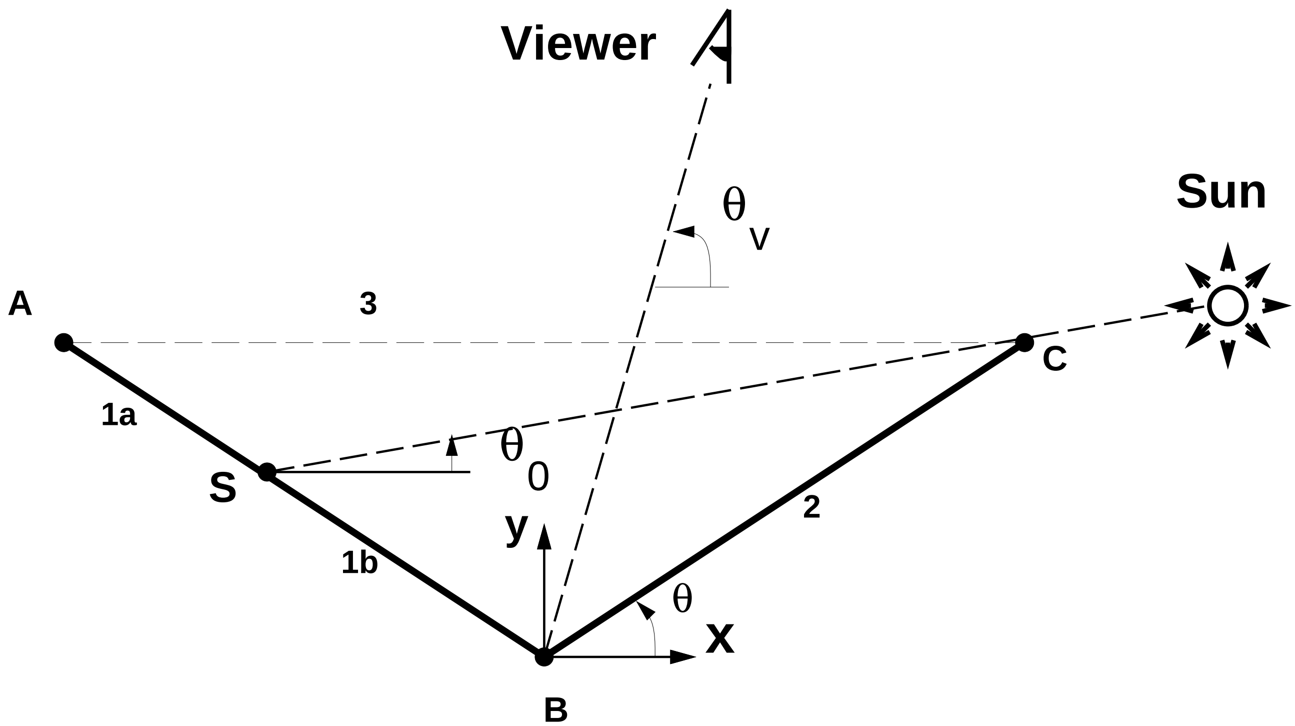

Figure 1: Geometry of a wedge

Wedge Geometry

We note that the points on the wedge are given by :
$\vec{A} = (-w\cos\theta, w\sin\theta)$, $\vec{B} = (0,0)$, $\vec{C} = (w\cos\theta, w\sin\theta)$, and the shadow point $\vec{S}$ is given by : $\vec{S} = (\vec{A} - \vec{B})f_p$ where :

$$f_p = \begin{cases} \frac{\sin\theta\cos\theta_0 - \cos\theta\sin\theta_0}{\sin\theta} & 0 < \theta_0 < \theta \\ 0 & \theta < \theta_0 < \frac{\pi}{2} \end{cases} .$$

String Lengths

Thus we are only considering illumination angles from $0 < \theta_0 < \frac{\pi}{2}$ so that shadows can only occur on facet 1. The slope angle is θ and the illumination angle θ_0 is measured from the horizon.
The distances between points or "string lengths" are given by :

$$\overline{AB} = |\vec{A} - \vec{B}| = w,$$
$$\overline{BC} = |\vec{B} - \vec{C}| = w,$$
$$\overline{AC} = |\vec{A} - \vec{C}| = 2w\cos\theta,$$
$$\overline{AS} = |\vec{A} - \vec{S}| = (1 - f_p)w,$$
$$\overline{BS} = |\vec{B} - \vec{S}| = f_p w,$$
$$\overline{CS} = |\vec{C} - \vec{S}|.$$

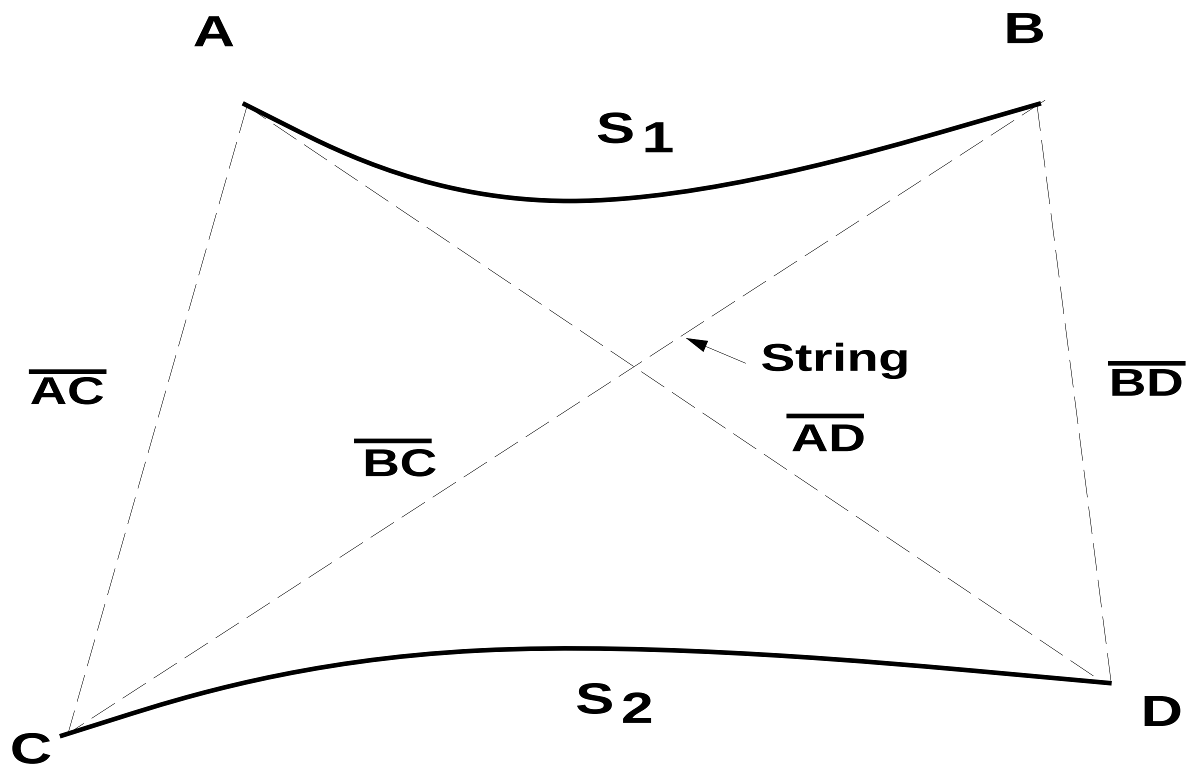

Figure 2: Crossed string method to compute view factors.

View Factors

The view factors can be computed using Hottel's crossed string method, Hottel and Sarofim (1967). An example of the method is shown in Fig. 2. The view factor between two infinite strips with areas S_1 and S_2 is given by :

$$S_1 F_{12} = \frac{\overline{AD} + \overline{BC} - \overline{AC} - \overline{BD}}{2} ,$$

where $\overline{AD}$ is the length between the points A and D, etc.

We list now the view factors between all facets ($1a$, $1b$, 2 and 3). Note that view factors from and to facet $1b$ exist only if there is a shadow point $\vec{S}$:

$$F_{1a2} = \frac{\overline{AB} + \overline{CS} - \overline{AC} - \overline{BS}}{2w(1-f_p)},$$

$$F_{1b2} = \frac{\overline{BS} + \overline{BC} - \overline{CS}}{2wf_p},\ if\vec{S}\ exists,$$

$$F_{21a} = \frac{\overline{AB} + \overline{CS} - \overline{AC} - \overline{BS}}{2w},$$

$$F_{21b} = \frac{\overline{BC} + \overline{BS} - \overline{CS}}{2w},\ if\vec{S}\ exists,$$

$$F_{1a3} = \frac{\overline{AS} + \overline{AC} - \overline{CS}}{2w(1-f_p)},$$

$$F_{1b3} = \frac{\overline{AB} + \overline{CS} - \overline{AS} - \overline{BC}}{2wf_p},\ if\vec{S}\ exists,$$

$$F_{31a} = \frac{\overline{AS} + \overline{AC} - \overline{CS}}{4w\cos\theta},$$

$$F_{31b} = \frac{\overline{AB} + \overline{CS} - \overline{AS} - \overline{BC}}{4w\cos\theta},\ if\vec{S}\ exists,$$

$$F_{23} = \frac{\overline{BC} + \overline{AC} - \overline{AB}}{2w},$$

and

$$F_{32} = \frac{\overline{BC} + \overline{AC} - \overline{AB}}{4w\cos\theta}.$$

Radiosity Equations for a Wedge

Facets $1a$ and 2 are illuminated and have initial emission terms :

$$E_{1a} = \rho_1 E_0 \cos(\frac{\pi}{2} - \theta_0 + \theta),\ if\ |\frac{\pi}{2} - \theta_0 + \theta| < \frac{\pi}{2}$$
$$E_2 = \rho_2 E_0 \cos(\frac{\pi}{2} - \theta_0 + \theta),\ if\ |\frac{\pi}{2} - \theta_0 + \theta| < \frac{\pi}{2}$$

where ρ_1 is the reflectance of slope 1 and ρ_2 is the reflectance of slope 2.
E_0 is the total incident solar power per unit area in $[Wm^{-2}]$.
The radiosity equations for the three facets can then be written as :

$$B_{1a} = E_{1a} + \rho_1[F_{1a2}B_2 + F_{1a3}B_{sky}],$$
$$B_{1b} = \rho_1[F_{1a2}B_2 + F_{1b3}B_{sky}],$$

and

$$B_2 = E_2 + \rho_2[F_{21a}B_{1a} + F_{21b}B_{1b} + F_{23}B_{sky}].$$

Solution of Radiosity Equations

The above equations can be solved iteratively and the convergence criterion is met when the root mean square error Δ of successive iterations $(i+1)$ and (i) reaches a lower limit ϵ :

$$\Delta^{(i+1)} = \sqrt{\frac{1}{3}[(B_{1a}^{(i+1)} - B_{1a}^{(i)})^2 + (B_{1b}^{(i+1)} - B_{1b}^{(i)})^2 + (B_{2}^{(i+1)} - B_{2}^{(i)})^2]},$$

for $i = 1, 2, 3, ...$

Note that the following weighted sums of view factors add up to unity :

$$F_{1a2}(1 - f_p) + F_{1b2} f_p + F_{1a3}(1 - f_p) + F_{1b3} f_p = 1,$$
$$F_{21a} + F_{21b} + F_{23} = 1,$$

and

$$F_{31a} + F_{31b} + F_{32} = 1.$$

Note : *We have included the contributions from sky light (B_{sky}) in these formulas but have set the term to zero for our calculations.*

BRF of a Wedge

To compute the BRF from the radiosities, we need to compute the visibility of each facet from a given view direction θ_v which is defined as the counter clockwise angle measured from the x axis. The BRF is then given in general by (see Borel and Gerstl (1994)) :

$$\rho_{radiosity}(\theta_0;\theta_v) = \frac{P_{1a}B_{1a} + P_{1b}B_{1b} + P_2 B_2}{E_0}.$$

The visibilities P_i for each facet depend on the slope and viewing angle and can be written as :

$$P_{1a} = \begin{cases} 1 & 0 < \theta_v < \theta_c \\ \frac{1-f_p}{1-f_v} & \theta_c < \theta_v < \theta \\ \frac{1-f_p}{2}[1 + \frac{\tan\theta}{\tan\theta_v}] & \theta < \theta_v < \pi - \theta \\ 0 & \pi - \theta < \theta_v < \pi \end{cases},$$

$$P_{1b} = \begin{cases} 0 & 0 < \theta_v < \theta_c \\ \frac{f_p - f_v}{1-f_v} & \theta_c < \theta_v < \theta \\ \frac{f_p}{2}[1 + \frac{\tan\theta}{\tan\theta_v}] & \theta < \theta_v < \pi - \theta \\ 0 & \pi - \theta < \theta_v < \pi \end{cases},$$

and

$$P_2 = \begin{cases} 0 & 0 < \theta_v < \theta \\ \frac{1}{2}[[1 - \frac{\tan\theta}{\tan\theta_v}] & \theta < \theta_v < \pi - \theta \\ 1 & \pi - \theta < \theta_v < \pi \end{cases}.$$

The angle θ_c is given by :

$$\theta_c = \tan^{-1}[\frac{C_y - S_y}{C_x - S_x}].$$

The variable f_v is given by :

$$f_v = \begin{cases} \frac{\sin\theta\cos\theta_v - \cos\theta sin\theta_v}{\sin\theta} & 0 < \theta_v < \theta \\ 1 - \frac{-\sin\theta\cos\theta_v - \cos\theta sin\theta_v}{\sin\theta} & \pi - \theta < \theta_v < \pi \end{cases}.$$

The projection of the ray from the observer onto the wedge is the :

$$\vec{V} = \begin{cases} (\vec{A} - \vec{B})f_v & 0 < \theta_v < \theta \\ (\vec{C} - \vec{B})f_v & \pi - \theta < \theta_v < \pi \end{cases}.$$

The sum of the visibilities is equal to unity for all view angles from 0 to π. The BRF of a wedge, neglecting multiple reflections between the facets, is given by :

$$\rho_{single}(\theta_0; \theta_v) = \frac{P_{1a}E_{1a} + P_{1b}E_{1b} + P_2E_2}{E_0}.$$

Results

The simple wedge model yields some interesting results which seem at first not correct but can be explained. For example if we compute the BRF for a wedge with a slope angle $\theta = 50^0$ and a sun angle of $\theta_0 = 40^0$ we get the BRF shown in Fig. 3.
The radiosity computed BRF for viewing zenith angles greater than 50^0 is larger than unity. How can this be? First we note that the light is perpendicularly incident on facet 1, thus the reflectance in the single scattering case must be equal to unity. Second we note that the shaded facet 2 when viewed at angles less than -40^0 is completly dark for the single scattering case but has a BRF of 0.2744 for a 100 % reflecting surface. Third we note that the view factor from facet $1a$ to facet 2 is $F_{1a2} = 0.3064$. Fourth, the amount of light facet $1b$ receives from facet 2 is given by $F_{1a2}B_2 = 0.30640.2744 = 0.0840$ which must be added to the single scattering term. Thus we have shown that the BRF for this case can be indeed be larger than unity. From Fig. 3 one can see that the effect of scattering of light from the adjacent facet is greatest in the shadowed region ($\theta_v < -40^0$), this is consistent with the results of other studies.

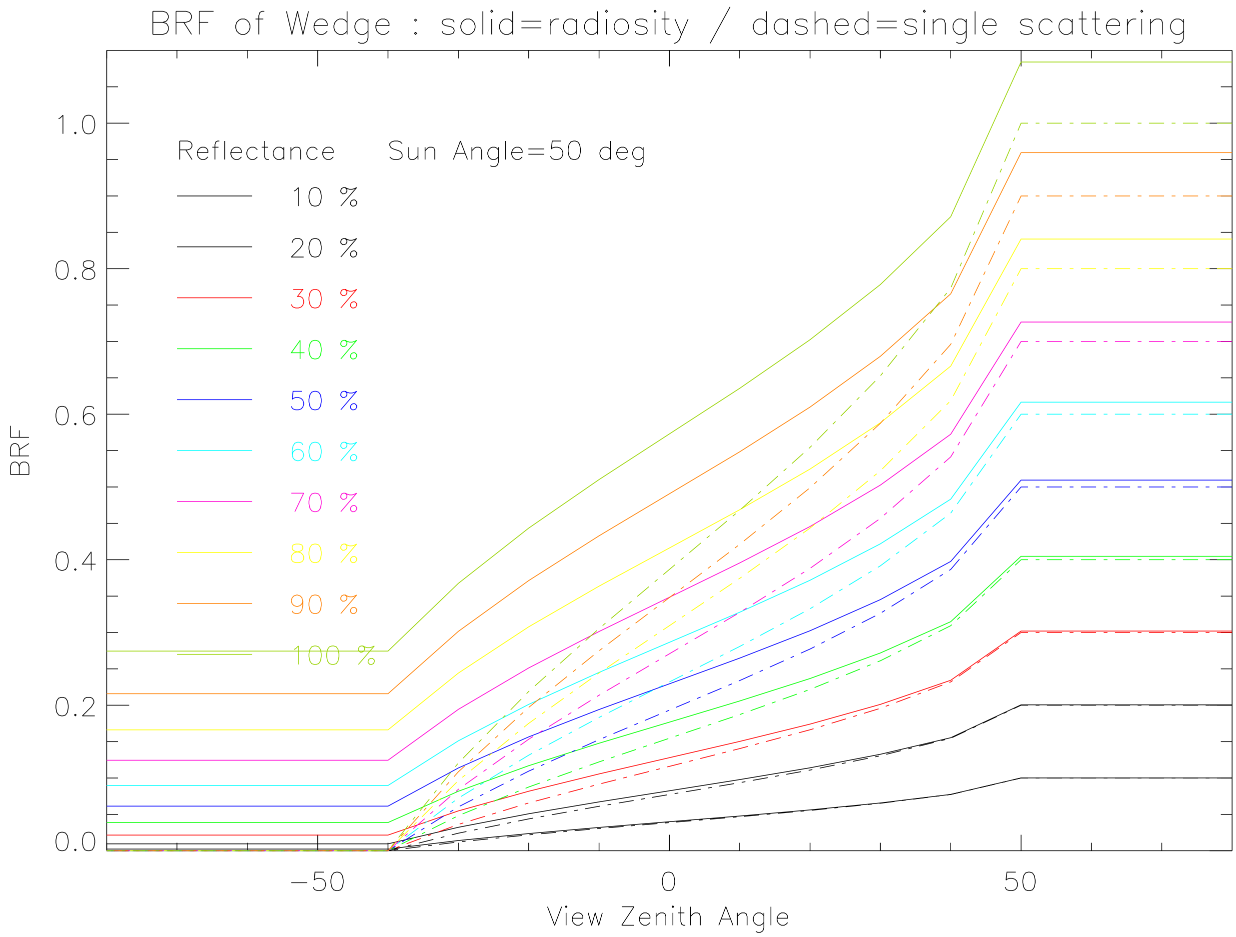

Figure 3: BRF of a wedge with a slope angle $\theta = 50^0$ and a sun angle of $\theta_0 = 40^0$

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